NASHVILLE STORMWATER MANAGEMENT MANUAL

VOLUME 3—THEORY

CHAPTER 3
Open Channel Hydraulics

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Chapter 3
OPEN CHANNEL HYDRAULICS

SYNOPSIS

The conveyance capacity of most drainage facilities can be evaluated using the principles of open channel hydraulics. This chapter provides introductory material on open channel terminology; mass, energy, and momentum principles; and critical, uniform, and nonuniform flow conditions. The fundamental relationship for performing open channel capacity calculations is Manning's Equation. Key references for information presented in this chapter are Chow (1959), Henderson (1966), and French (1985). Supplemental references include Streeter (1971), Simon (1981), Rouse (1950), and USDOT, FHWA (1961) and (1965).

3.1 TERMINOLOGY

An open channel is defined as any conduit conveying a fluid in which the liquid surface is exposed to the atmosphere as a boundary. Pipe flow occurs in a conduit that is closed to atmospheric pressure and subject to hydraulic pressure alone. Open channel conditions are the basis for most stormwater management calculations, except for some storm sewer or culvert systems. Pipe flow fundamentals are discussed in Chapters 5 (Culverts) and 6 (Storm Sewers).

Open channel flow problems can be more complicated than similar problems in pressure pipes. This is primarily because physical conditions (such as cross section, slope, and roughness) are typically more variable in channels. Calculations for open channel flow problems tend to be more empirical than those for pipes, and there is greater uncertainty when assigning friction factors.

Terminology important to an understanding of open channel flow problems is briefly defined and discussed below.

3.1.1 GEOMETRIC ELEMENTS

Open channel flow problems generally require an evaluation of various geometric elements associated with the shape of the channel. For most artificial or constructed open channels, geometric elements can be determined mathematically in terms of depth of flow and other dimensions for the channel shape. For most natural channel sections, however, profile sections based on the actual variations in the depth of flow across the section are generally required. The following geometric terminology is pertinent to the fundamentals of open channel hydraulics:

Prismatic channel. An artificial channel with non-varying cross section and constant bottom slope.

Channel section. The cross section of a channel taken perpendicular to the direction of flow.

Depth of flow. The vertical distance from the lowest point of a channel section to the free surface, designated as y. The depth of flow measured perpendicular to the channel bottom is known as the depth of flow for the section and is designated as d. When the channel slope is small (less than 1 percent), the depths y and d are essentially equal. The relationship between y and d is expressed as:

$$y = \frac{d}{\cos\theta} \qquad (3\text{-}1)$$

where:

y = Vertical depth of flow, in feet

d = Perpendicular depth of flow, in feet

θ = Slope angle of the channel bottom, in degrees

Stage. The elevation or vertical distance of the water surface relative to a datum. If the lowest point of a channel is taken as the datum, then the stage and depth of flow are equal.

Control section. Any section at which the depth of flow is known or can be controlled at a given stage for specified discharge rates.

Top width. The width of the channel section at the free surface.

Water area. The cross-sectional area of the flow perpendicular to the direction of flow.

Wetted perimeter. The length of the line of intersection of the channel wetted surface with a cross-sectional plane perpendicular to the direction of flow.

Hydraulic radius. The ratio of the water area to its wetted perimeter, which is expressed mathematically as:

$$R = \frac{A}{P} \tag{3-2}$$

where:

R = Hydraulic radius of the channel, in feet

A = Water area of the channel; in square feet

P = Wetted perimeter of the channel, in feet

Hydraulic depth. The ratio of the water area to top width, which is expressed mathematically as:

$$d_m = \frac{A}{T} \tag{3-3}$$

where:

d_m = Hydraulic depth or mean depth of flow, in feet

A = Water area of the channel, in square feet

T = Top width of the channel, in feet

Equivalent depth. The depth corresponding to an area of flow having a width twice the depth, expressed mathematically as:

$$d_e = (A/2)^{1/2} \tag{3-4}$$

where:

d_e = Equivalent depth, in feet

A = Cross-sectional flow area, in square feet

The equivalent depth is often used to calculate the Froude number for flow in pipes under open channel flow conditions.

Critical flow section factor. The product of the water area and the square root of the hydraulic depth, expressed mathematically as:

$$Z = A\ (\sqrt{d_m})^{1/2} = A\ \frac{A}{T}^{1/2} \qquad (3\text{-}5)$$

where:

Z = Critical flow section factor

A = Water area, in square feet

d_m = Hydraulic depth (see Equation 3-3), in feet

T = Top width of channel, in feet

Uniform flow section factor. Based on Manning's Equation, the product of the water area and the hydraulic radius raised to the 2/3 power, expressed mathematically as:

$$U_F = AR^{2/3} \qquad (3\text{-}6)$$

where:

U_F = Uniform flow section factor for Manning's Equation

A = Water area, in square feet

R = Hydraulic radius, in feet

3.1.2 STEADY/UNSTEADY FLOW

Time variations of open channel flow can be classified as either steady or unsteady. Steady flow occurs in an open channel when the discharge or rate of flow at any location along the channel remains constant with respect to time. The maintenance of steady flow in any channel reach requires that the rates of inflow and outflow be constant and equal. Conversely, open channel flow is unsteady when the discharge at any location in the channel changes with respect to time. During periods of stormwater runoff, the inflow hydrograph to an open channel is usually unsteady. However, in practice, open channel flow is generally assumed to be steady at the discharge rate for which the channel is being designed (i.e., peak discharge of the inflow hydrograph).

3.1.3 UNIFORM/NONUNIFORM FLOW

Spatial variations of open channel flow can be classified as either uniform or nonuniform.

Uniform flow occurs only in a channel of constant cross section, slope, and roughness, known as a uniform open channel. If a given channel segment is uniform, the mean velocity and depth of flow will be constant with respect to distance. When the requirements for uniform flow are met, the depth of flow for a given discharge is defined as the normal depth of flow. In practice, minor variations in the channel bottom or deviations from the average cross section can be ignored as long as the average values are representative of actual channel conditions.

A general assumption is that uniform flow is also steady flow. It is reasonable to conclude that when the water surface remains constant with time, depth will remain constant with distance.

True uniform flow rarely exists in either natural or artificial channels. Any change in the channel cross section, slope, or roughness with distance causes the depths and average velocities to change with distance. Flow that varies in depth and velocity when the discharge is constant, or steady, is defined as steady nonuniform flow.

Unsteady nonuniform flow, in which there are variations of both space and time, is the most complex type to evaluate mathematically. Chow (1959), Henderson (1966), or French (1985) should be consulted for theoretical information.

Nonuniform flow may be further classified as either rapidly or gradually varied. Rapidly varied flow is also known as a local phenomenon, examples of which include the hydraulic jump and hydraulic drop. The primary example of gradually varied flow occurs when subcritical flow is restricted by a culvert or storage reservoir. The water surface profile caused by such a restriction is generally referred to as a backwater curve. Additional theoretical information on nonuniform flow evaluations is presented in Section 3.5.

3.1.4 LAMINAR/TURBULENT FLOW

The effect of fluid viscosity relative to the inertial forces of motion is an important property to consider when evaluating open channel flow.

Laminar flow generally occurs when the viscous forces are strong relative to inertial forces. Water particles will appear to move in definite smooth paths, or streamlines, when flow is laminar. Laminar flow is known to occur in shallow overland or sheet flow conditions.

When the viscous forces are weak relative to the inertial forces, the flow can be classified as turbulent. In turbulent flow, the water particles move in irregular paths that are neither smooth nor fixed, and the result is a random mixing motion. Because turbulent flow is the most common type occurring in open channel facilities, it is the type considered for most hydraulic procedures, excluding shallow overland flow.

Operational limits for laminar and turbulent flow can be evaluated using a dimensionless parameter known as the Reynolds number, which is expressed mathematically as:

$$Re = \frac{vL}{\nu} \tag{3-7}$$

where:

Re = Reynolds number, dimensionless

v = Average velocity of flow, in feet/second

L = Characteristic length, in feet (Hydraulic radius as presented in Equation 3-2)

ν = Kinematic viscosity of fluid, in square feet/second

Since viscosity is in the denominator, low Reynolds number values are associated with laminar flow (high viscosity relative to inertial forces) and high Reynolds number values are associated with turbulent flow (high inertial forces relative to viscosity).

Numerous experiments have been performed to establish operational limits of the Reynolds number that will define when laminar and turbulent flow occur. When flow can be classified as neither laminar nor turbulent, it is called transitional. When the hydraulic radius is used to determine the characteristic length in Equation 3-7, then the following operational limits of the Reynolds number are reported (French, 1985):

Laminar Flow	$Re \leq 500$
Transitional Flow	$500 \leq Re \leq 12{,}500$
Turbulent Flow	$Re \geq 12{,}500$

3.1.5 SUBCRITICAL/CRITICAL/SUPERCRITICAL FLOW

The importance of gravity as a driving force in open channel drainage systems makes its effect on the state of flow a major factor for evaluation. This can be done using a dimensionless parameter known as the Froude number, which is expressed mathematically as:

$$Fr = \frac{v}{\sqrt{gL}} \tag{3-8}$$

where:

Fr = Froude number, dimensionless

v = Average velocity of flow, in feet/second

g = Acceleration due to gravity, 32.2 feet/second^2

L = Characteristic length, in feet (Hydraulic depth as presented in Equation 3-3)

The Fr value is the dimensionless ratio of inertial forces to gravity forces. If Fr values are less than 1, gravity forces dominate and the open channel is said to be operating in the subcritical range of flow. This is sometimes called tranquil flow and is characterized as relatively deep, low velocity flow with respect to critical flow. Depth of flow can be controlled at a downstream location.

If Fr values are greater than 1, inertial forces dominate and the open channel is said to be operating in the supercritical range of flow. This is also called rapid or shooting flow and is characterized as relatively shallow, high velocity flow with respect to critical flow. Depth of flow can be controlled at an upstream location.

When the Fr value equals 1, inertial forces and gravity forces are balanced and the open channel exhibits critical flow. Additional information on critical flow conditions is presented in Section 3.3.

3.2 MASS, ENERGY, AND MOMENTUM

The following three basic principles are generally applied to open channel flow evaluations:

1. Conservation of mass
2. Conservation of energy
3. Conservation of linear momentum

3.2.1 MASS

The conservation of mass or continuity equation for continuous steady flow can be expressed mathematically as:

$$Q = Av \qquad (3\text{-}9)$$

where:

Q = Discharge, in cfs

A = Cross-sectional area, in square feet

v = Average channel velocity, in feet/second

For continuous unsteady flow, the continuity equation must include time as a variable. Additional information on the conservation of mass for unsteady flow can be obtained from Chow (1959), Henderson (1966), or French (1985).

3.2.2 ENERGY

It can be useful at times to consider the total energy head of an open channel. Because energy input must equal output, the total energy head of two points in a channel reach will equal one another. This equality is commonly known as the energy equation, which is expressed as:

$$d_1 \cos\theta + \frac{v_1^2}{2g} + z_1 = d_2 \cos\theta + \frac{v_2^2}{2g} + z_2 + h_{loss} \qquad (3\text{-}10)$$

where:

d_1 and d_2 = Perpendicular depths of flow at channel sections 1 and 2, respectively, in feet

θ = Slope angle of the channel bottom, in degrees

v_1 and v_2 = Average velocities at channel sections 1 and 2, respectively, in feet/second

g = Acceleration due to gravity, 32.2 feet/second^2

z_1 and z_2 = Bottom elevations above an arbitrary datum at channel sections 1 and 2, respectively, in feet

h_{loss} = Head or energy loss between channel sections 1 and 2, in feet

For small channel slopes (less than 1 percent), the Energy Equation is expressed as:

$$y_1 + \frac{v_1^2}{2g} + z_1 = y_2 + \frac{v_2^2}{2g} + z_2 + h_{loss} \qquad (3\text{-}11)$$

where:

y_1 and y_2 = Vertical depths of flow at channel sections 1 and 2, respectively, in feet

v_1 and v_2 = Average velocities at channel sections 1 and 2, respectively, in feet/second

g = Acceleration due to gravity, 32.2 feet/second²

z_1 and z_2 = Bottom elevations above an arbitrary datum at channel sections 1 and 2, respectively, in feet

h_{loss} = Head or energy loss between channel sections 1 and 2, in feet

When the head loss between channel sections is zero, either Equation 3-10 or 3-11 can be referred to as the Bernoulli equation.

Equations 3-10 and 3-11 ignore that the actual velocity distribution over a channel section is nonuniform (i.e., slow along the bottom and higher in the middle). When nonuniform velocity distributions are considered, the velocity head is generally greater than the value computed using the average channel velocity. Kinetic energy coefficients that can be used to account for nonuniform velocity conditions at bridges are discussed in Chapter 7.

For typical prismatic channels with a fairly straight alignment, the effect of disregarding the existence of a nonuniform velocity distribution is negligible, especially when compared to other uncertainties involved in such calculations. Therefore, Equations 3-10 and 3-11 are appropriate for most open channel problems. However, if velocity distributions are known or suspected to be non-typical, velocity coefficient adjustments should be considered.

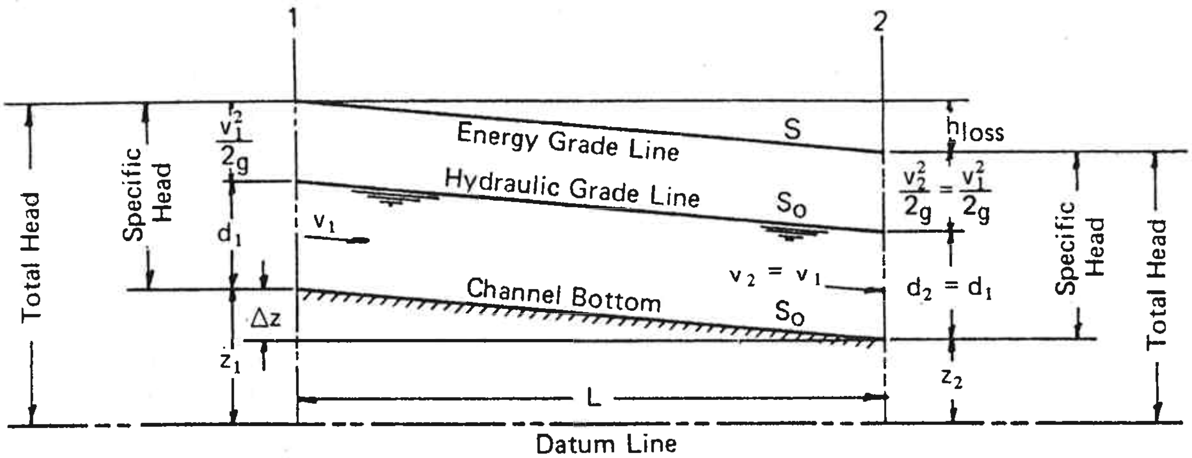
Equations 3-10 and 3-11 also assume that the hydrostatic law of pressure distribution is applicable. This law states that the distribution of pressure over the channel cross section is the same as the distribution of hydrostatic pressure; that is, that the distribution is linear with depth. The assumption of a hydrostatic pressure distribution for flowing water is valid only if the flow is not accelerating or decelerating in the plane of the cross section. Thus, these equations should be restricted to conditions of uniform or gradually varied nonuniform flow. If the flow is known to be rapidly varying, other relationships should be considered (e.g., momentum).

A longitudinal profile of total energy head elevations is called the energy grade line (gradient). The longitudinal profile of water surface elevations is the hydraulic grade line (gradient). The energy and hydraulic grade lines for uniform open channel flow are illustrated in Figure 3-1. For flow to occur in an open channel, the energy grade line must have a negative slope in the direction of flow. A gradual decrease in the energy grade line for a given length of channel represents the loss of energy caused by friction. When considered together, the hydraulic and energy grade lines reflect not only the loss of energy by friction, but also the conversion between potential and kinetic forms of energy.

For uniform flow conditions, the energy grade line is parallel to the hydraulic grade line, which is parallel to the channel bottom (see Figure 3-1). Thus, for uniform flow, the slope of the channel bottom becomes an adequate basis for the determination of friction losses. During uniform flow, no conversions occur between kinetic and potential forms of energy. If the flow is accelerating, the hydraulic grade line would be steeper than the energy grade line, while retarding flow would produce an energy grade line steeper than the hydraulic grade line.

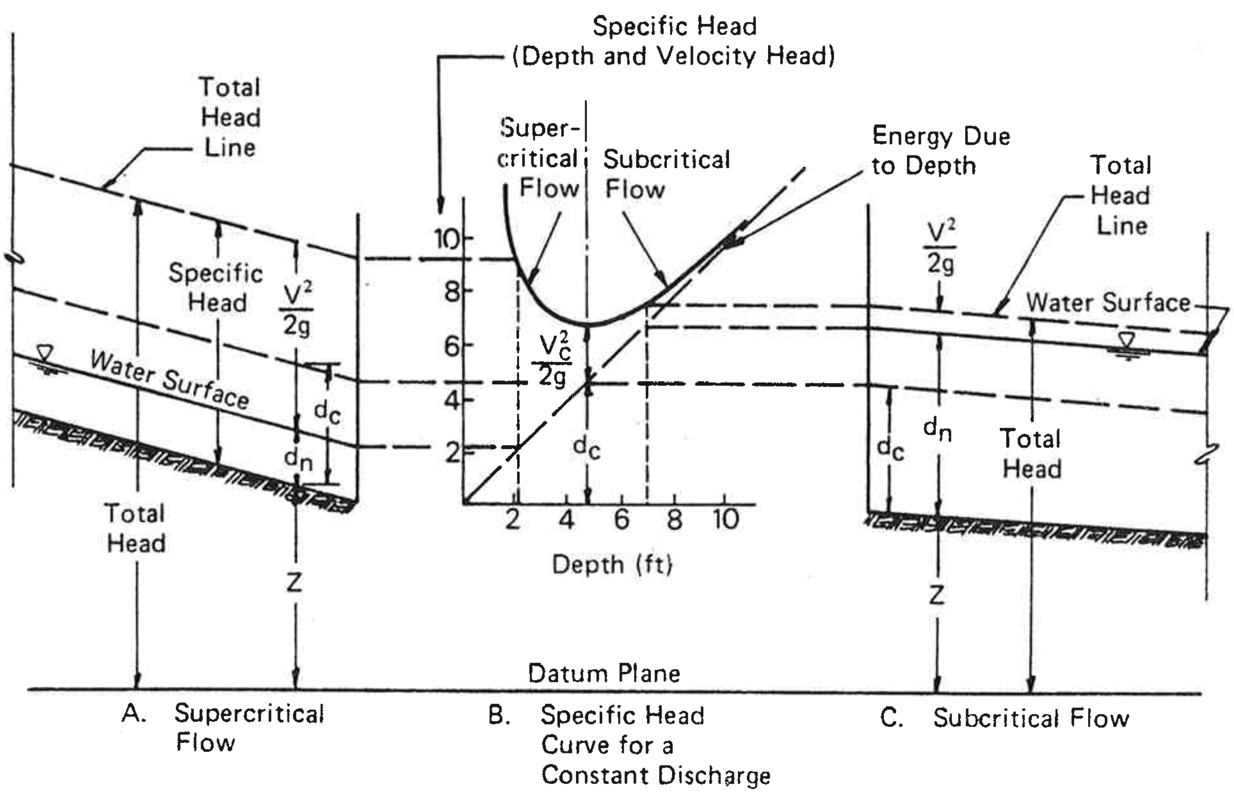
3.2.3 MOMENTUM

According to Newton's Second Law of Motion, the change of momentum per unit of time is equal to the resultant of all external forces applied to the moving body. Application of this principle to open channel flow produces a relationship that is virtually the same as the energy equation expressed in Equations 3-10 and 3-11. In theory, the two principles are unique, primarily because energy is a scalar quantity (magnitude only), while momentum is a vector quantity (magnitude and direction). In addition, the head loss



Reference: USDOT, FHWA, HDS-3 (1961).

FIGURE 3-1
Characteristics of Uniform Open Channel Flow



Reference: USDOT, FHWA, HDS-3 (1961).

FIGURE 3-2
Definition Sketch for Specific Head and Subcritical and Supercritical Flow

determined by the energy equation measures the internal energy dissipated in a particular channel reach, while the momentum equation measures the losses caused by external forces exerted on the water by the walls of the channel. However, for uniform flow, since the losses caused by external forces and internal energy dissipation are equal, the momentum and energy equations give the same results.

Application of the momentum principle has certain advantages for problems involving high changes of internal energy, such as a hydraulic jump. Thus, the momentum principle should be used to evaluate rapidly varied nonuniform flow conditions. The hydraulic jump is a common method for accomplishing energy dissipation at culvert, storm sewer, and channel outlets. Energy dissipation is covered in Chapter 10.

3.3 CRITICAL FLOW

The energy content of flowing water with respect to the channel bottom is often referred to as the specific energy head, which is expressed by the equation:

$$E = d + \frac{v^2}{2g} \tag{3-12}$$

where:

E = Specific energy head, in feet

d = Depth of open channel flow, in feet

v = Average channel velocity, in feet/second

g = Acceleration due to gravity, 32.2 feet/second^2

Considering the relative values of potential energy (depth) and kinetic energy (velocity head) in an open channel can greatly aid the hydraulic analysis of open channel flow problems. These analyses are usually performed using a curve showing the relationship between the specific energy head and the depth of flow for a given discharge in a given channel that can be placed on various slopes. The curve representing specific energy head for an open channel is generally used to identify regions of supercritical and subcritical flow conditions. This information can be helpful for performing hydraulic capacity calculations and evaluating the suitability of channel linings and flow transition sections.

3.3.1 SPECIFIC ENERGY AND CRITICAL DEPTH

A typical curve representing the specific energy head of an open channel is illustrated in Figure 3-2, Part B. The straight diagonal line on this figure represents points where the depth of flow and specific energy head are equal. At such points the kinetic energy is zero; therefore, this diagonal line is a plot of the potential energy, or energy due to depth. The ordinate interval between the diagonal line of potential energy and the specific energy curve for the desired discharge is the velocity head, or kinetic energy, for the depth in question. The lowest point on the specific energy curve represents flow with the minimum content of energy. The depth of flow at this point is known as the critical depth. The general equation for determining the critical depth is expressed as:

$$\frac{Q^2}{g} = \frac{A^3}{T} \qquad (3\text{-}13)$$

where:

Q = Discharge, in cfs

g = Acceleration due to gravity, 32.2 feet/second^2

A = Cross-sectional area, in square feet

T = Top width of water surface, in feet

Critical depth for a given channel can be calculated through trial and error with Equation 3-13. However, in practice, it may be computed more easily by using either the equations for selected channel cross sections or the critical depth charts presented in Volume 2. Chow (1959) presents a procedure for the analysis of critical flow that uses the critical flow section factor, Z, as defined by Equation 3-5.

Using the definition of the critical section factor and a velocity distribution coefficient of one, the equation for critical flow conditions is:

$$Z = \frac{Q}{\sqrt{g}} \qquad (3\text{-}14)$$

where:

Z = Critical flow section factor (see Equation 3-5)

Q = Discharge, in cfs

g = Acceleration due to gravity, 32.2 feet/second^2

When the discharge is known, Equation 3-14 gives a value for the critical section factor, allowing Equation 3-5 to be solved for the critical depth. Conversely, when the critical section factor is known, the discharge can be calculated by rearranging Equation 3-14.

The determination of critical depth is independent of the channel slope and roughness, because critical depth represents a depth for which the specific energy head is a minimum. According to Equation 3-13, the magnitude of critical depth depends only on the discharge and the shape of the channel. Thus, any given size and shape of channel has only one critical depth for the given discharge, which is independent of the channel slope or roughness. However, if Z is not a single-valued function of depth, it is possible to have more than one critical depth. For a given value of specific energy, the critical depth results in the greatest discharge, or conversely, for a given discharge, the specific energy is a minimum for the critical depth.

3.3.2 CRITICAL VELOCITY

The velocity at critical depth is called the critical velocity. An equation for determining the critical velocity in an open channel of any cross section is expressed as:

$$v_c = \sqrt{g\ d_m} \qquad (3\text{-}15)$$

where:

v_c = Critical velocity, in feet/second

g = Acceleration due to gravity, 32.2 feet/second^2

d_m = Mean depth of flow or equivalent depth, d_e, for pipes, in feet, calculated from Equation 3-3 or 3-4

3.3.3 SUPERCRITICAL FLOW

For conditions of uniform flow, the critical depth, or point of minimum specific energy, occurs when the channel slope equals the critical slope (i.e., the normal depth of flow in the channel is critical depth). When channel slopes are steeper than the critical slope and uniform flow exists, the specific energy head is higher than the critical value, because of the higher values of the velocity head (kinetic energy). This characteristic of open channel flow is illustrated by the specific head curve segment to the left of critical depth in Figure 3-2, Part B, and is known as supercritical flow. Supercritical flow is characterized by relatively shallow depths and high velocities, as shown in Figure 3-2, Part A. If the natural depth of flow in an open channel is supercritical, the depth of flow at any point in the channel may be influenced by an upstream control section. The relationship of supercritical flow to the specific energy curve is shown in Figure 3-2, Parts A and B.

3.3.4 SUBCRITICAL FLOW

When channel slopes are flatter than the critical slope and uniform flow exists, the specific energy head is higher than the critical value, because of the higher values of the normal depth of flow (potential energy). This characteristic of open channel flow is illustrated by the specific head curve segment to the right of critical depth in Figure 3-2, Part B, and is known as subcritical flow. Subcritical flow is characterized by relatively large depths with low velocities, as shown in Figure 3-2, Part C. If the natural depth of flow in an open channel is subcritical, the depth of flow at any point in the channel may be influenced by a downstream control section. The relationship of subcritical flow to the specific energy curve is shown in Figure 3-2, Parts B and C.

3.3.5 SPECIFIC ENERGY CONSIDERATIONS

Several points about Figure 3-2 should be noted. First, at depths of flow near the critical depth for any discharge, a minor change in specific energy will cause a much greater change in depth. Second, velocity head for any discharge in the subcritical portion of the specific energy curve in Figure 3-2, Parts A and B, is relatively small when compared to specific energy. For this subcritical portion of the specific energy curve, changes in depth of flow are approximately equal to changes in specific energy. Finally, the velocity head for any discharge in the supercritical portion of the specific energy curve increases rapidly as depth decreases. For this supercritical portion of the specific energy curve, changes in depth are associated with much greater changes in specific energy.

3.4 UNIFORM FLOW

Although steady uniform flow is rare, it is practical in many cases to assume that uniform flow occurs in appropriate segments of an open channel system. The results obtained from calculations based on this assumption will be approximate and general, but can often provide satisfactory solutions.

3.4.1 MANNING'S EQUATION

The hydraulic capacity of an open channel is usually determined through application of Manning's Equation, which determines the average velocity when given the depth of flow in a uniform channel cross section. Given the velocity, the capacity, Q, is calculated as the product of velocity and cross-sectional area (see Equation 3-9).

Manning's Equation is an empirical equation in which the values of constants and exponents have been derived from experimental data for turbulent flow conditions. According to Manning's Equation, the mean velocity of flow is a function of the channel roughness, the hydraulic radius, and the slope of the energy gradient. As noted previously, for uniform flow, the slope of the energy gradient is assumed to be equal to the channel bottom slope. Manning's Equation is expressed mathematically as:

$$v = \frac{1.49}{n} R^{2/3} S^{1/2} \qquad (3\text{-}16)$$

or

$$Q = \frac{1.49}{n} AR^{2/3} S^{1/2} \qquad (3\text{-}17)$$

where:

v = Average channel velocity, in feet/second

Q = Discharge, in cfs

n = Manning's roughness coefficient

R = Hydraulic radius of the channel, in feet, calculated using Equation 3-2

S = Slope of the energy gradient, in feet/foot

A = Cross-sectional area of the open channel, in square feet

A detailed discussion of procedures for solving Manning's Equation is presented in Volume 2.

3.4.2 BEST HYDRAULIC SECTION

The best hydraulic section of an open channel can be determined mathematically by using the continuity equation of open channel flow (Equation 3-9) and Manning's Equation (Equation 3-17). According to the continuity equation, if the cross-sectional area is to be a minimum, the velocity must be a maximum for any given cross-sectional area. According to Manning's Equation, the velocity is a maximum for a given cross section and channel slope when the hydraulic radius is a maximum. The hydraulic radius is a maximum when the wetted perimeter is minimized for a given cross-sectional area (see Equation 3-2).

It can be shown mathematically that the hydraulic cross section that maximizes the velocity, and thus minimizes the area required to convey a given discharge, is a semicircle (see Streeter, 1971, or Chow, 1959). Therefore, the best trapezoidal hydraulic section is one that approximates a semicircle (see Figure 3-3, Part a). For the special case in which the trapezoid is a rectangle, the best shape is that for which the width is twice the depth (see Figure 3-3, Part b).

In practice, the best hydraulic section of an open channel may be altered from the mathematically ideal cross section to account for the following factors:

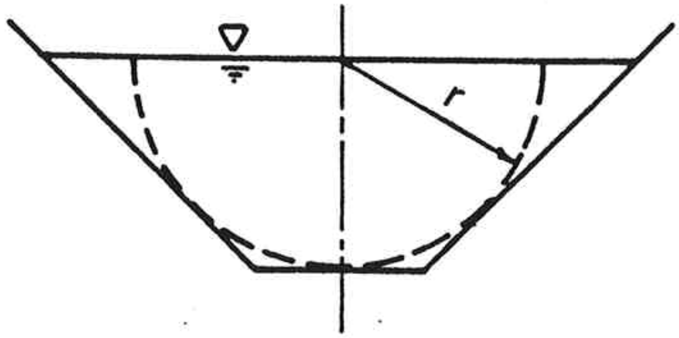
1. The average velocity of the best hydraulic section should not exceed the maximum permissible velocity for the channel bottom in question.

2. The best hydraulic section may not produce the minimum total excavation if a significant overburden must be removed.

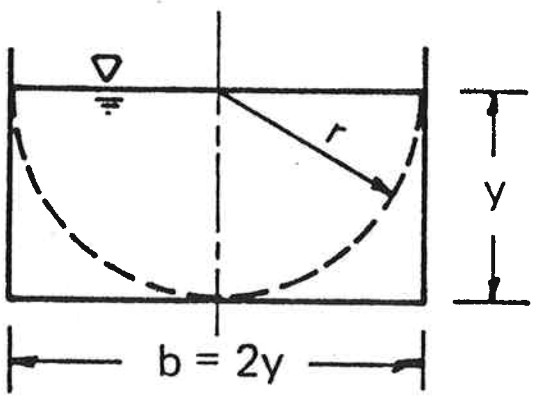
3. The proportions of an open channel may vary widely without significantly changing the required hydraulic elements.

4. The cost of excavation is not solely dependent on the amount of material removed. Considerations such as the ease of access and disposal may be more important than the volume of material excavated.

5. The method of construction or available equipment that will be used can affect the type of excavation.



(a) Trapezoidal



(b) Rectangular

FIGURE 3-3
Best Hydraulic Sections for Trapezoidal and Rectangular Channels

3.5 NONUNIFORM FLOW

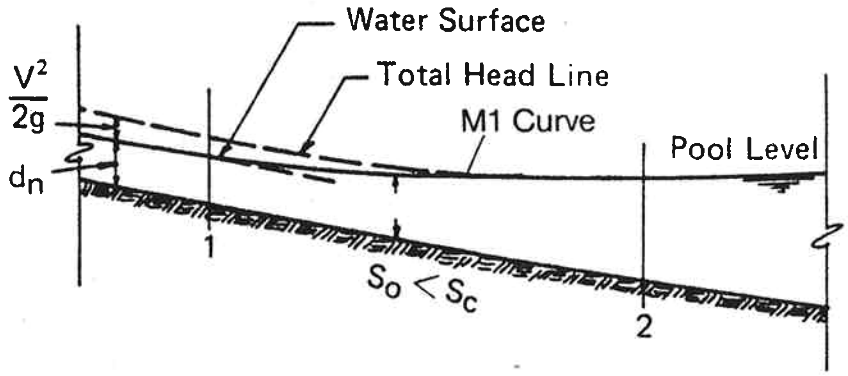
In the vicinity of changes in the channel section or slope that will cause nonuniform flow profiles, the direct solution of Manning's Equation is not possible, since the energy gradient for this situation does not equal the channel slope. Three typical examples of nonuniform flow are illustrated in Figures 3-4 through 3-6. The following sections describe these nonuniform flow profiles and briefly explain how the total head line is used for approximating these water surface profiles in a qualitative manner. For gradually varied flow, the direct step and standard step methods for performing backwater calculations are discussed. The hydraulic jump is discussed for rapidly varied flow.

3.5.1 GRADUALLY VARIED FLOW

A channel on a mild slope (subcritical) discharging into a reservoir or pool is illustrated in Figure 3-4. The vertical scale is exaggerated for clearer illustration.

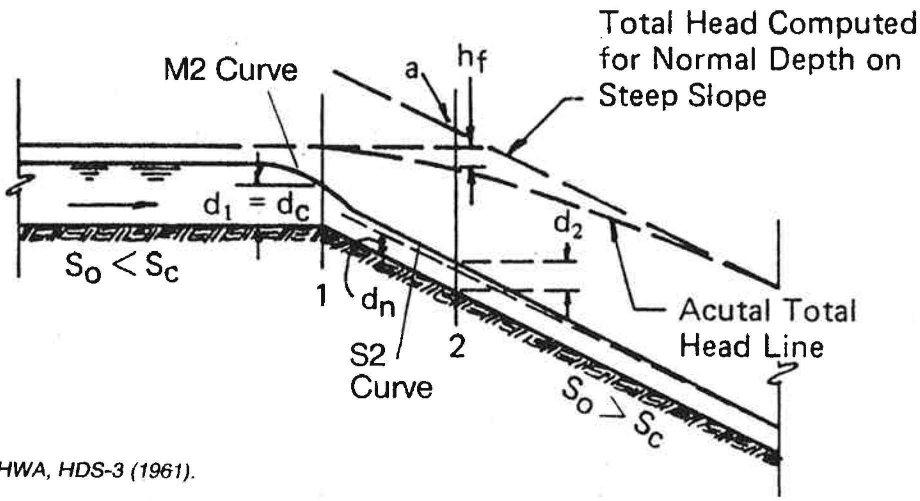
Cross Section 1 is located upstream of the pool, where uniform flow occurs in the channel, and Cross Section 2 is located at the beginning of a level pool. The depth of flow between Sections 1 and 2 is changing, and the flow is nonuniform. The water surface profile between the sections is known as a backwater curve referred to as an M1 curve, and is characteristically very long.

A channel in which the slope changes from subcritical (mild) to supercritical (steep) is illustrated in Figure 3-5. The flow profile passes through critical depth near the break in slope (Section 1). This is true whether the upstream slope is mild, as in the sketch, or the water above Section 1 is ponded, as would be the case if Section 1 were the crest of a spillway of a dam. If, at Section 2, the total head were computed, assuming normal depth on the steep slope, it would plot (Point a in Figure 3-5) above the elevation of total head at Section 1. This is physically impossible, because the total head line must slope downward in the direction of flow. The actual total head line will take the position shown and have a slope approximately equal to S_c at Section 1 and approaching S_o farther downstream. The drop in the total head line, h_{loss}, between Sections 1 and 2 represents the loss in energy due to friction.



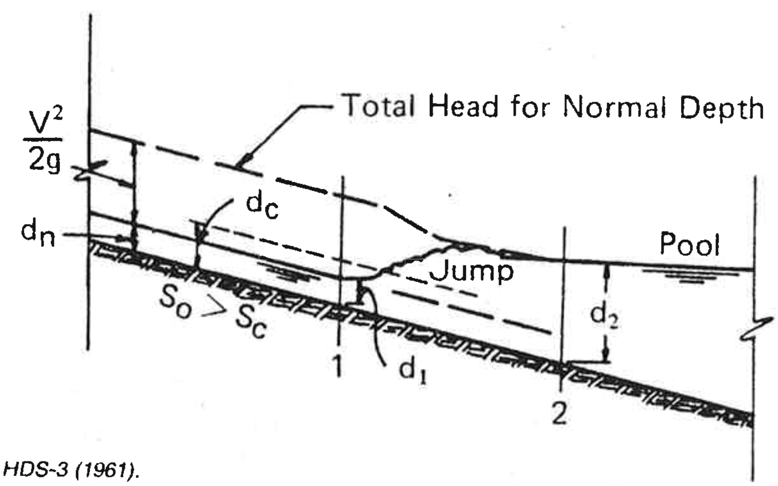
Reference: USDOT, FHWA, HDS-3 (1961).

FIGURE 3-4
Nonuniform Water Surface Profile for Downstream Control Caused by a Flow Restriction



Reference: USDOT, FHWA, HDS-3 (1961).

FIGURE 3-5
Nonuniform Water Surface Profile Caused by a Change in Slope Conditions



Reference: USDOT, FHWA, HDS-3 (1961).

FIGURE 3-6
Nonuniform Water Surface Profile Caused by a Hydraulic Jump

At Section 2, the actual depth, d_2, is greater than normal depth, d_n, because sufficient acceleration has not occurred, and the assumption of normal depth at this point would clearly be in error. As Section 2 is moved downstream, so that total head for normal depth drops below the pool elevation above Section 1, the actual depth quickly approaches the normal depth for the steep channel. This type of water surface curve (Section 1 to Section 2) is referred to as an S2 curve and is characteristically much shorter than the M1 backwater curve discussed previously.

Another common type of nonuniform flow is the drawdown curve to critical depth that occurs upstream from Section 1 (Figure 3-5) where the water surface passes through critical depth. The depth gradually increases upstream from critical depth to normal depth, provided that the channel remains uniform over a sufficient distance. The length of the drawdown curve, referred to as an M2 curve, is much longer than the curve from critical depth to normal depth in the steep channel.

Direct Step Method

For prismatic channels, when the resistance coefficient and shape are constant with distance, gradually varied water surface profiles can be calculated using the direct step method. This method calculates a water surface profile by determining the distance between cross sections with specified flow depths. The friction slope can be calculated using the following equation:

$$S_f = \frac{n^2 \ v^2}{2.22 \ R^{4/3}} \tag{3-18}$$

where:

S_f = Friction slope, in feet/foot

n = Manning's n value

v = Average velocity, in feet/second

R = Hydraulic radius, in feet

The length of channel between sections with specified flow depths is calculated as:

$$\Delta x = \frac{\Delta E}{S_o - S_f} \tag{3-19}$$

where:

Δx = Channel distance between sections with specified flow depths, in feet

ΔE = Change in specific energy between sections (depth plus velocity head), in feet

S_o = Bottom slope, in feet/foot

S_f = Friction slope, in feet/foot

A step-by-step procedure, tabulation form, and example problem for the direct step method are presented in Volume 2.

Standard Step Method

A procedure suitable for calculating gradually varying water surface profiles when the channel cross section and resistance coefficient vary with distance is the standard step method. This method provides an estimate of the depth of flow at specified longitudinal distances.

The friction slope can be calculated using Equation 3-18. The friction loss in a specified channel reach length is calculated as:

$$h_f = \bar{S}_f \Delta x \tag{3-20}$$

where:

h_f = Friction loss for a reach length of Δx, in feet

$\bar{S}_f$ = Average friction slope between sections for reach length Δx, in feet/foot

Δx = Channel reach length, in feet

The eddy loss, which can be significant in non-prismatic channels, can be calculated using the following equation:

$$h_e = k_e \frac{v^2}{2g} \tag{3-21}$$

where:

h_e = Eddy losses, in feet

k_e = Eddy loss coefficient

v = Average velocity, in feet/second

g = Acceleration due to gravity, 32.2 feet/second

For gradually converging and diverging reaches, k_e is 0 to 0.1 and 0.2, respectively. For abrupt expansions and contractions, k_e is about 0.5. For prismatic and regular channels, $k_e = 0$ (Chow, 1959).

A trial and error procedure based on an assumed depth is required to develop the water surface profile. The correct solution for a given section is obtained when the total head based on velocity and depth is equal to the total head based on friction losses (Equation 3-10) and eddy losses (Equation 3-21).

A step-by-step procedure, tabulation form, and example problem for the standard step method are presented in Volume 2.

3.5.2 RAPIDLY VARIED FLOW

A hydraulic jump occurs when a steep (supercritical) channel discharges into a reservoir or pool. This special case condition is illustrated in Figure 3-6. A hydraulic jump makes a dynamic transition from the supercritical flow in the steep channel to the subcritical flow in the pool. This situation differs from that shown in Figure 3-4 in that the flow approaching the pool in Figure 3-6 is supercritical and the total head in the approach channel is large, relative to the pool depth. In general, supercritical flow can be changed to subcritical flow only by passing through a hydraulic jump. The violent turbulence in the jump dissipates energy rapidly, causing a sharp drop in the total head line between the supercritical and subcritical states of flow. A jump will occur whenever the ratio of the depth in the approach channel, d_1, to the depth in the downstream channel, d_2, reaches a specific value. In Figure 3-6, normal depth in the approach channel persists beyond the point where the projected pool level would intersect the water surface of the channel at normal depth. Normal depth can be assumed to exist on the steep slope upstream from Section 1, which is located at about the toe of the jump.

The relationship between variables for a hydraulic jump can be expressed as:

$$\frac{d_2}{d_1} = \frac{\left(1 + 8Fr^2{}_1\right)^{0.5} - 1}{2} \qquad (3\text{-}22)$$

where:

d_2 = Depth below jump, in feet

d_1 = Depth above jump, in feet

Fr_1 = Froude number above jump, dimensionless (see Equation 3-8)

Additional discussion on the use of hydraulic jumps to dissipate energy is presented in Chapter 10. Detailed information on the quantitative evaluation of hydraulic jump conditions in open channels is available in publications by Chow (1959), Henderson (1966), French (1985), and Streeter (1971), and in HEC-14 from USDOT, FHWA (1983). In addition, handbooks by Brater and King (1976) and the USDA, SCS (NEH-5, 1956) may be useful.

3.6 CHANNEL STABILIZATION

3.6.1 IMPROVEMENTS

While quantification of the effects of channel improvements is difficult, qualitative assessment of possible impacts and appropriate action are required for all channel modifications. The proportionality given below provides a basis for qualitative assessment of stream modifications; the two sides will adjust to maintain a constant proportion.

$$QS \propto Q_s d_{50} \tag{3-23}$$

where:

Q = Typical or dominant discharge, in cfs

S = Energy slope, in feet/foot

Q_s = Sediment transport, in pounds/second

d_{50} = Sediment mean particle diameter, in feet

Thus, if a check dam is built across a stream, Q_s decreases. Assuming Q and d_{50} remain constant, the slope must decrease to maintain the proportionality. The result is scour downstream from the dam and deposition upstream.

If a bend in a stream is straightened or a rough channel is paved, the slope is steepened. If S increases, either Q_s or d_{50} or both must also increase. The result is generally advancing scour upstream, higher velocities through the reach with associated channel deterioration, scour just below the reach, and eventual deposition downstream.

A tributary carrying excess sediment resulting from construction will tend to block the main channel, raising flow levels upstream and causing deposition downstream. The cause is an increased Q_s in the reach, with a resulting increase in S in the downstream direction.

Urbanization tends to increase the amount of water flowing in a channel for any given storm frequency (i.e., Q increases). A channel in Nashville tends to adjust its size to carry the 2- to 5-year storm within its banks. Thus, urbanization tends to cause not only increased flooding but increased channel size through bed bank erosion.

The sediment transport capacity of the stream will be approximately constant throughout the stream when neither aggradation nor deposition occurs. Bars may form and disappear and the outside of bends may show caving, but the general form of the stream is a constant. An estimate of sediment transport capacity in coarse sand and gravel bed streams can be made from the Meyer-Peter and Mueller formula (Simons and Senturk, 1977), expressed as:

$$Q_s = 600W (\tau - \tau_c) \qquad (3\text{-}24)$$

where:

Q_s = Bedload sediment discharge, in tons/day

W = Channel width, in feet

τ = Channel shear = γ RS, in pounds/square foot

γ = Specific weight of water, 62.2 pounds/cubic foot

R = Hydraulic radius, in feet

S = Channel slope, in feet/foot

τ_c = Critical shear = $4.82d_{50}$, in pounds/square foot

d_{50} = Sediment mean particle diameter, in feet

This formulation assumes that the sediment has a specific gravity of 2.65, which is common for quartz rock. Additional information on channel aggradation and degradation can be found in Chapter 7.

3.6.2 RIPRAP

Riprap design will be influenced by one of three different types of flow conditions. In Type I flow, the velocity in the upstream channel is higher than in the riprap-lined reach. Riprap size will be determined by the faster upstream velocity. The stone roughness can be expected to slow the velocity enough within a transition length of about 75 to 100 times d_{50} to design for Type II flow conditions for the remainder of the reach.

Type II flow occurs when the velocity in the riprap-lined reach is expected to be higher than the upstream channel velocity. A typical example is a broad, deep channel discharging into a shallow, narrow reach. Designing riprap for Type II flow is an iterative process. A stone size and associated roughness value are assumed, velocity calculated based on the roughness value, and a stone size determined. The calculations are repeated until the assumed and computed stone sizes are reasonabley close.

Type III flow is encountered when the turbulence acting to dislodge the stone is not generated by general boundary shear but by abrupt changes in boundary geometry. In such cases, a general riprap design procedure is not possible. Application of experienced engineering judgment, specific designs (e.g., stilling basin design), or the selection of conservative parameters are possible options.

The procedure given in Volume 2 is an adaptation of basic research performed by S. T. Maynord (1987) at the Waterways Experiment Station and developmental work by A. J. Reese (1984 and 1988). The original equation developed by Maynord (1987) is expressed as:

$$d_{30}/D = C \left[\left(\frac{\gamma}{\gamma_s - \gamma} \right)^{0.5} \frac{v}{(gD)^{0.5}} \right]^{2.5} \qquad (3\text{-}25)$$

where:

d_{30} = Diameter of stone for which 30 percent, by weight, of the gradation is finer, in feet

D = Depth of flow above stone, in feet

v = Mean point velocity above the stone, in feet/second

γ_s = Specific weight of stone, in pounds/cubic foot

γ = Specific weight of water, 62.4 pounds/cubic foot

g = Acceleration due to gravity, 32.2 feet/second^2

The C factor is 0.30 for the bottoms of straight prismatic channels and both bottoms and sides of curved channels. The depth is the depth to the top of rock at the point in question (usually the toe of the side slope) and the velocity is the mean point velocity. The C factor for side slopes of straight prismatic channels is 0.24. With a safety factor, a general constant is given as 0.36, yielding a 1.2 safety factor for the former case and 1.5 for the latter. The C factor is a constant when d_{30} is used as the representative stone size but varies with d_{85}/d_{15} when d_{50} is used.

If it is assumed that the specific weight of the stone is 165 (a typical value for Nashville), the following equation is developed:

$$d_{30}/D = 0.193 \left(\frac{v}{\sqrt{gD}}\right)^{2.5} \qquad (3\text{-}26)$$

where:

d_{30} = Diameter of stone for which 30 percent, by weight, of the gradation is fine, in feet

D = Depth of flow above stone, in feet

v = Mean point velocity above the stone, in feet

g = Acceleration due to gravity, 32.2 feet/second²

Since d_{50} is the size typically recognized in design, the d_{30} size from Equation 3-26 can be converted to d_{50} by assuming that for typical ranges of d_{85}/d_{15} (2-2.3), $d_{50} = 1.20\ d_{30}$.

Correction factors for a different specific weight and for placement in a bend were derived through the use of the concept of <u>effective velocity</u>. The correction factors are used to establish an effective velocity when site-specific conditions deviate from the base condition. The correction factors actually represent a ratio of the effective velocity and the actual velocity. The base conditions are for thickness equal to d_{100} and mean point velocity in a straight channel.

To correct for different specific weights, the following equation is used:

$$C_g = \left(\frac{102.6}{(\gamma_s - 62.4)}\right)^{0.5} \qquad (3\text{-}27)$$

where:

C_g = Correction coefficient for specific weight

γ_s = Specific weight of selected riprap, in pounds/cubic foot

A graphical solution to this equation is presented in Volume 2. Equation 3-26 requires using a point velocity over the toe. For straight channels, an adequate estimate of point velocity is the average velocity in the main portion of the channel. This is different from the mean velocity for the entire channel cross section when flow exceeds the banks of the main channel (e.g., flood plains) and if flow is slowed considerably out to about 0.5 times the main channel depth. Thus, in most cases, the channel should be broken into segments or some rule of thumb used to adjust the mean channel velocity. The ratio of mean channel velocity to point mean velocity over the toe for prismatic channels ranges from 1.1 to 1.3 and for natural channels from 1.3 to 1.5. In all cases, the greater the bank-bed roughness difference and relative flow area above the side slopes, the greater the adjustment required.

A second factor is required to account for the special conditions encountered in a bend. Typically, the point velocity in a bend is about 1.1 to 1.5 times the average velocity for a straight channel section including overbank flow. The U.S. Army Corps of Engineers (1970) provide information from various sources concerning the velocity increase for bends. A graphical approach is presented in Volume 2.

Maynord (1987) also showed that, for a range of d_{85}/d_{15} from 2.0-2.32, a thickness of stone greater than the typical d_{100} could allow for a smaller stone size. A graphical relationship to account for this adjustment is presented in Volume 2.